L I T

Investment Technology Consulting

The Lambda equity return-generating model

Generating time-dependent real return probabilities for major equity markets is a key input to the LAMbda Advice Manager: a suite of decision tools integrating resource planning, risk tolerance discovery and dynamic portfolio optimization. Optimal equity portfolios are customized to personal risk utility functions by combining risky assets with a risk-free asset specific to the nature of the target and its time horizon. The separation of major equity markets, with evidenced return histories, from all other assets with uncertain outcomes leads to a parsimonious asset mix relative to conventional mean variance optimization approaches which rely implicitly on diversification of risky assets to manage risk. With risk controlled by the amount of risk-free asset exposure, does the parsimonious risky asset set lead to better forecasts of portfolio risk and return?

1. Modeling directly from real returns

Because LAMbda was designed for individuals, not institutional funds like pension schemes and life companies, LIT's focus was always on modeling real return probabilities, to inform decisions about preservation of real capital values or the achievement of real spendable income or real growth in wealth.

To make the best projections of real asset values, LIT chose to model equity real returns directly from historical real returns. Any decision process could then compare these uncertain real return probabilities with known market yields for appropriate safe harbour assets: index linked gilts (guaranteed by the Government to pay a particular fixed real return whatever happens to nflation) or shorter term index linked National Savings & Investment Certificates.

Modeling directly from past real returns puts the entire process on the foundation of 'systematic' behaviour. A system is defined as obeying a particular set of rules from which it gains a degree of predictability or dependability. However, in this case the foundation is only as firm as the opinion that the evidence that markets tend to grow at a sustainable trend rate in real terms is a genuine instance of 'reversion to the mean' and not just a coincidence.

The strength of this belief can be drawn from a combination of observation and statistical tests over many different periods and of different markets - but it cannot be proven. It can also be drawn from a view of an underlying 'equilibrium' theory or model, be it of the economy itself (and so mirrored in trends for real income growth, productivity or real house prices) or of the return process itself (perhaps because expected risk premiums relative to safe harbour returns are bounded by some relatively constant 'risk utility function' for the 'average investor').

Mean reversion is widely but by no means universally accepted by investment professionals and is implicit in most projections for equity-based products, including the illustrative growth rates the FSA requires product providers to use.

In models in which equity returns are modelled indirectly from known market yields (realistic for valuation and accounting uses), the relatively predictable information contained in knowledge of the past trend of achieved real equity return gets lost in a myriad of unstable and largely random interdependencies. It is normally some theory of ‘risk premiums’ that forms the link between market yields for fixed income and equity returns but the equity risk premium process is highly unstable.

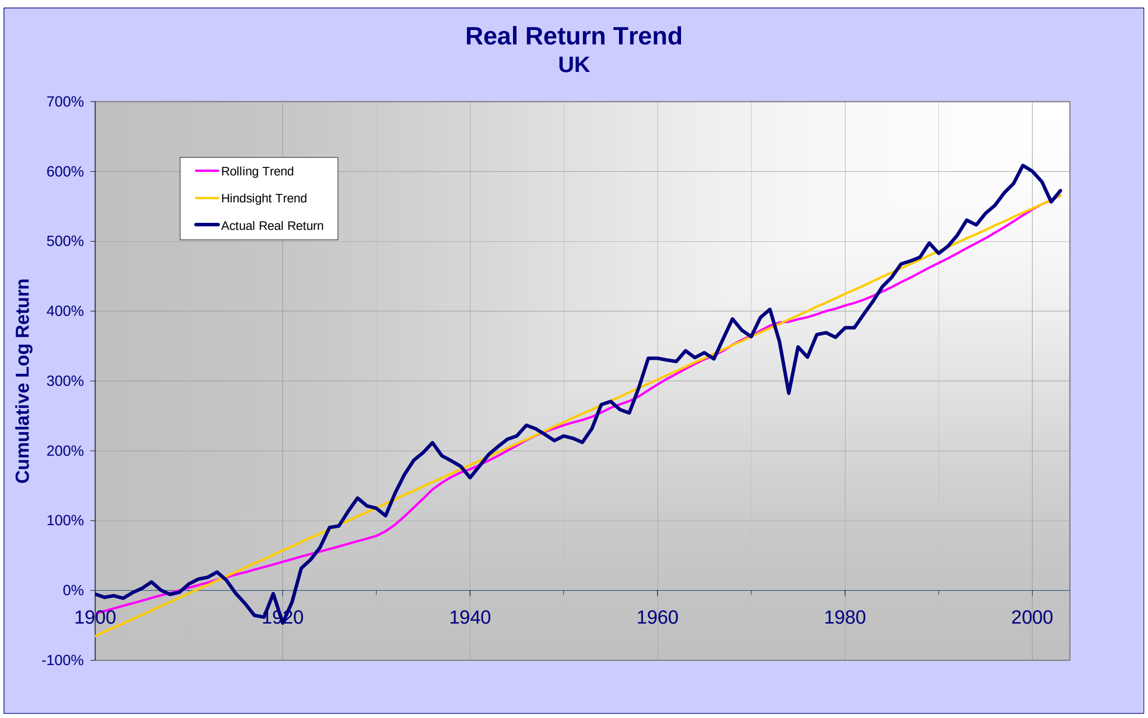
LIT’s view is that this produces inferior projections and unhelpfully unstable messages over the course of a journey about where the investor is in relation to their target. Methods appropriate for accounting and valuing liabilities, which should use discount rates derived from the market at the time, are not the most appropriate for generating return projections for equities.

The quantity that shows the features of an equilibrium system is *real total return* for a whole market. They must be inflation-adjusted returns and they must be total returns, not just capital or income returns.

2. UK Equities

The graph below shows an index of cumulative, log, real, total returns (reinvesting gross income monthly) for UK equities. Using log returns keeps the actual deviations proportionally equal at any point in time.

Two trends (lines of best fit or regressions) are shown: the yellow trend uses all the data available now (so incorporating ‘hindsight’) and the pink trend uses data known only at the time.



Deviations above or below the pink trend would be the basis of any contemporaneous attempt to estimate where the market was in relation to trend. Deviations above imply higher-than-trend achieved returns and lower-than-trend future returns – and vice versa. Confidence about the information content of a naïve prediction can and should take into account the amount of data.

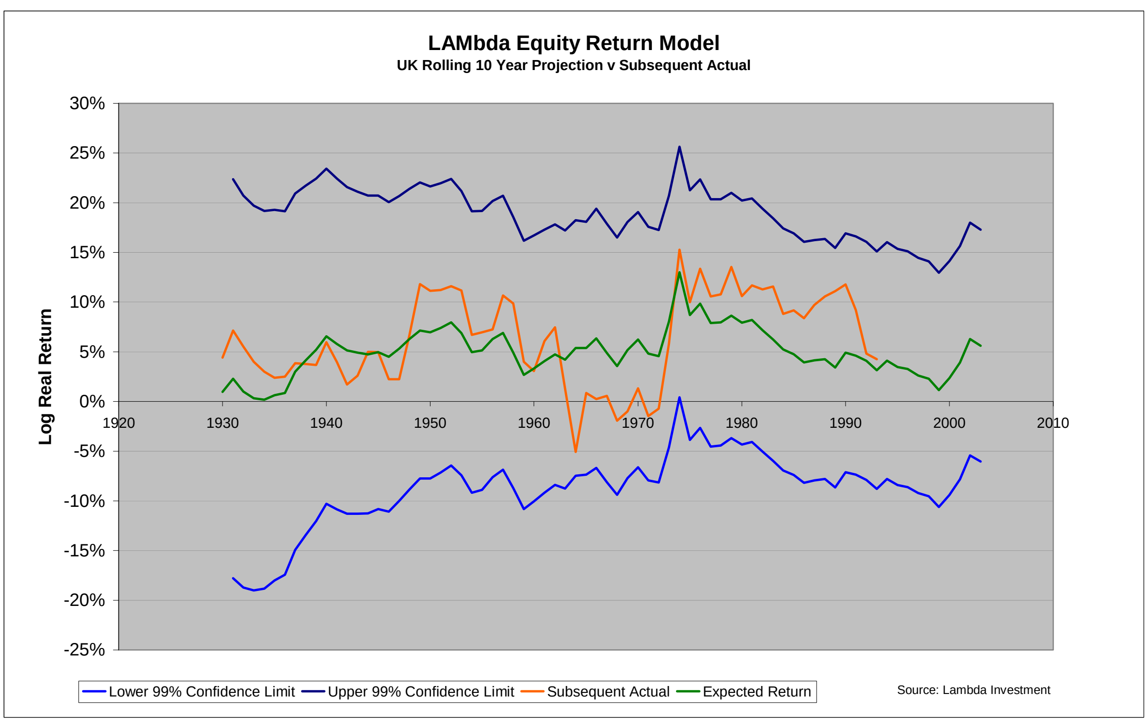
The term naïve is well chosen. Most model builders or model users would resist using only the information in the past trend, preferring to try to ‘explain’ these deviations by reference to factors such as interest rates, the growth of the economy, corporate profits or dividend yields and dividend growth. However, these attempts to fine-tune long-term predictions based on an interpretation of recent data fall into the trap that we don't know enough either about the behaviour of the economy or about the extent to which expectations about the economy are already reflected in the market price of equities.

Forecasters have not been good at either short-term or long-term equity return projections. It does not embarrass us at all to admit this collective failure and we have no expectation it will ever improve!

A naïve approach has the benefit it can be consistently followed. Since we are not trying to pick the most likely point outcome but rather quantify the distribution of probable outcomes, the key test of the naïve forecast is whether it produces a realistic range anchored on a realistic average. This can be seen for the UK market in the following graph.

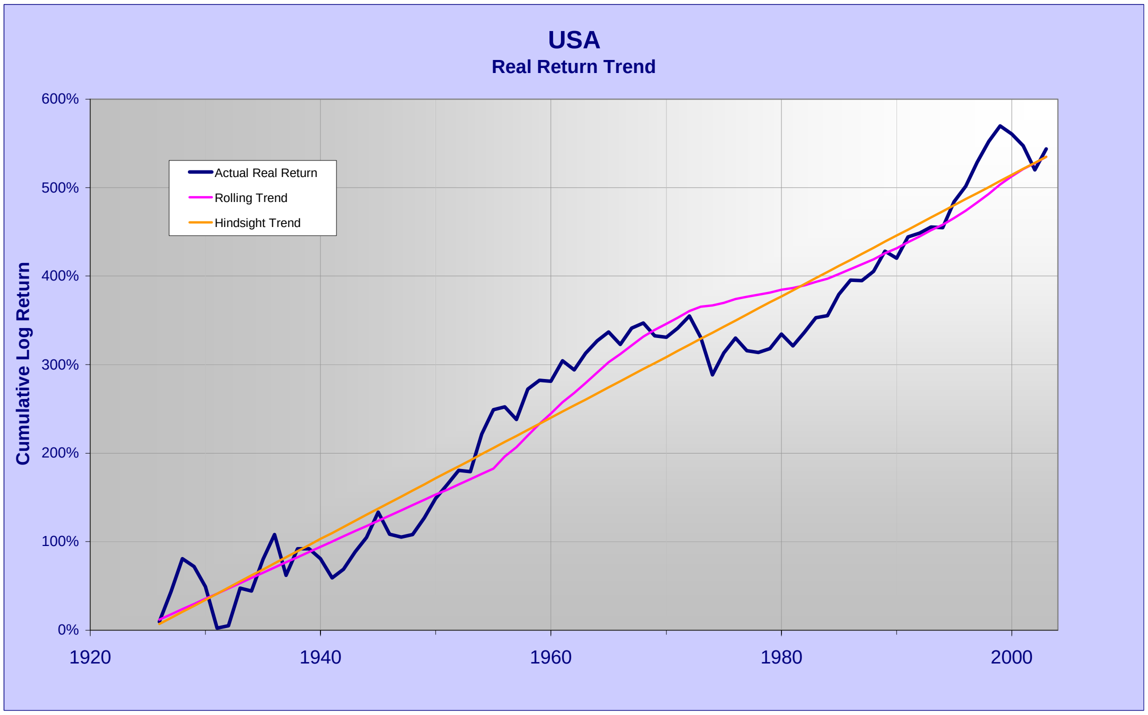
The green line in this graph is the mean of the predicted range of annualised future returns for a 10-year time horizon, made at each point without the benefit of hindsight. The orange line is the actual outcome. The two shades of blue mark the outer limits (at 99% certainty) of the predicted range as the LAMbda model is calibrated. The narrowing of the range over time reflects the additional confidence associated with additional data in the history.

We can see that projected returns much below 5% pa (green) were followed by returns well below 5% pa (orange) early in the period and at the end of the 1960s. We know that the low projected returns at the peak of the bull market in 1999 were followed by a correction but we have to wait to see whether a low 10-year return was correctly predicted (the orange line stops 10 years back). The average predicted return has been volatile around 5% pa but over the whole period the best fit is a growth rate of 6.1% since 1900. This is consistent with the 'normalized' real growth in many actuarial models used today, as we might expect since we all study the same data!

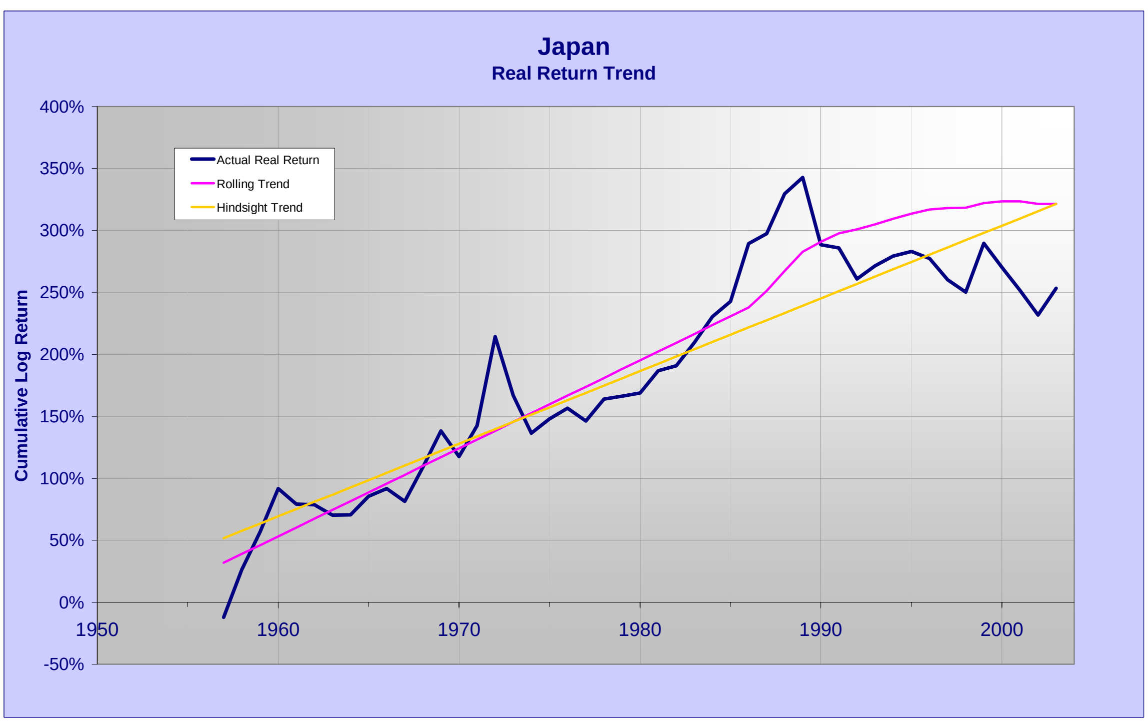


3. International Equities

Other equity markets' returns (in local currency terms, as experienced by domestic investors) show the same systematic features and even quite similar achieved growth trends.



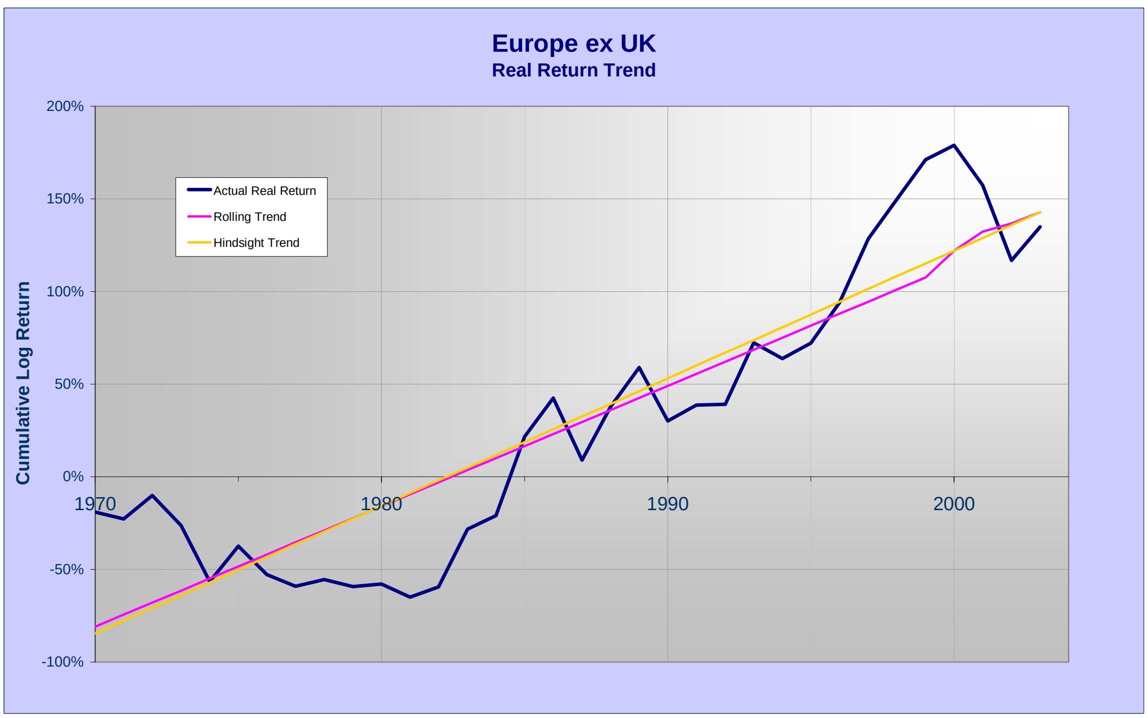
The US data for the S&P Index runs for nearly a century and over the whole period the growth trend for the same measure of real total return is 6.9% pa. This is the number the Lambda model defaults to when estimating a new range of projected returns, taking into account the initial deviation and adjusting for the length of data. Longer data from the 1870s shows a similar growth



trend but the data is less reliable.

For Japan, the notable difference is the amount of data. Pre-WWII data exists but there is little continuity due to the widespread recapitalisation of Japanese companies shortly after the war. Data starting in 1957 (so arguably omitting any post-war exceptional growth) shows a trend of 5.9% and this is what LIT assumes. The data adjustment, which increases the standard deviation of projected returns, is much larger for Japan than for the US.

We have the least data for Europe (excluding the UK) which is in any case a group of markets that have only become relatively homogenous in recent years, with increasing integration of trade and capital flows and increasing currency alignment. The trend of 6.9% is close to the other markets' longer term trends but this could be coincidental rather than symptomatic of a global real equity model with comparable investor risk preferences. The effect of having less reliable data is to increase the uncertainty LAMbda attributes to its projections for European equities.



4. Currencies

International equity projections also need to allow for uncertainty arising from currency movements against sterling. In a real return model, inflation differences and currency movements can realistically be assumed to cancel each other out over the long term. But over the short term there will be currency movements that are not explained by inflation differences. This unexplained currency volatility is smaller than and independent of the variance in market returns relative to trend. This considerably weakens the incremental risk associated with currency exposure.

The exchange rate trend explained by inflation differences, often referred to as 'purchasing power parity' or 'the real exchange rate', could be predicted by a model builder, based on the same idea of random deviations being bounded by mean reversion. Whilst the Lambda model can project currency returns using this approach, taking into account the starting deviation, LIT is not convinced there is enough data since exchange rates started to float wildly to be sure the real exchange rate and the starting deviation from it can be estimated sensibly. Model users may instead take into account the change in risk but not the change in returns explained by the additional exposure to exchange rate changes.

5. One model, different projections

The assumption underlying the model is that all these four major markets or market blocs behave *essentially* the same, showing bounded deviations from broadly similar long term trends. We do not need to know whether this arises because they are part of the same system, with global movements of capital contributing to the system features, or because they are different systems sharing similar average investor risk preferences.

But this underlying similarity of behaviour does not mean they behave similarly all the time. Though the graphs show many cyclical movements overlapping there are notable exceptions, such as Japan peaking in the late 1990s long after other markets, largely ignoring the 1987 crash, and then falling throughout the decade of the 1990s when other markets moved to high deviations. Sometimes, the direction is the same between several markets but the extent of the deviation varies, such as the UK reaching a more extreme undervaluation in 1974 than other markets.

These differences are important in providing opportunities to diversify equity risk and to take advantage from time to time of exceptional return opportunities but both are often exaggerated by professional investors. Perhaps the price of reasonable predictability within an equity market arising from this global equilibrium model is that they actually move together more than asset allocators would like. This can be seen by plotting for each market the deviations from hindsight-free trends as a ratio (referred to here as a ‘market value ratio’). A view about the degrees of co-movement becomes necessary when deciding how to use these return probabilities to construct portfolios and project portfolio-specific paths and outcomes, either for resource-planning purposes or for continuous portfolio management.